Feb-11-2016

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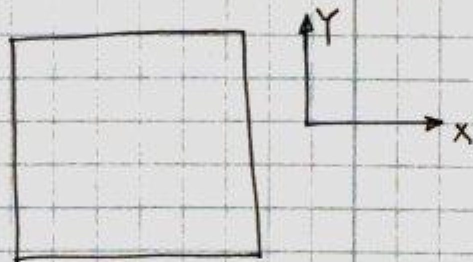
Exact constraint design → 2D

① **Design simple planar-exact constraint system:**

Plane - 2D — rigid body.

In the x-y plane, there are 3-'dof'.



Exact constraint — from Dr. Hale's PHD thesis

Great conceptualization of constraints.

- Statements:

① points on the object along the constraint line can move only at right angles to the constraint line, not along it.

② Any constraint along a given constraint line is functionally equivalent to any other constraint along the same constraint line (for small motions)

③ Any pair of constraints whose constraint lines intersect at a given point, is functionally equivalent to any other pair in the same ~~sp~~ plane whose constraint lines intersect at the same point. This is true for small motions and where the two constraints lie on distinctly different constraint lines.

④ The axes of a body's rotational degrees of freedom will each intersect all constraints applied to the body.

⑤ A constraint applied to a body removes that rotational degree of freedom about which it exerts a moment.

✓ Rule of thumb for planar problems: arrange constraint lines to form an equilateral triangle.

⑥ Any set of constraints, whose constraint lines intersect a complete and independent set of rotational axes is functionally equivalent to any other set of constraints whose constraint lines intersect the same or equivalent set of rotational axes. This is true for small motions and when each set ~~constraint~~ contains the same no. of independent constraints.

(7) An ideal sheet fixture imposes absolutely rigid constraint in its own plane. (X, Y and θ_Z), but it allows three degrees of freedom: Z, θ_X, θ_Y.

(8) An ideal wire flexure imposes absolutely rigid constraint along its axis (X), but it allows five degrees of freedom Y, Z, θ_X, θ_Y, θ_Z.

(9) A constraint (C) properly applied to a body (ie, without overconstraint) has the effect of removing one of the body's rotational degrees of freedom (R). The R removed is the one about which the constraint ~~app~~ exerts a moment. A body constrained by n constraints will have $6-n$ rotational degrees of freedom; each positioned such that no constraint exerts a moment about it.
In other words, each R will intersect all C's.

(10) Any pair of intersecting rotational degrees of freedom (R's) is equivalent to any other pair intersecting the same point and lying in the same plane. This holds true for small motions.

(11) Two parallel Rs are equivalent to any two parallel Rs, parallel to the first pair and lying in the same plane. They are also equivalent to a single R parallel to the first pair and lying in the same plane; and a C perpendicular to that plane.

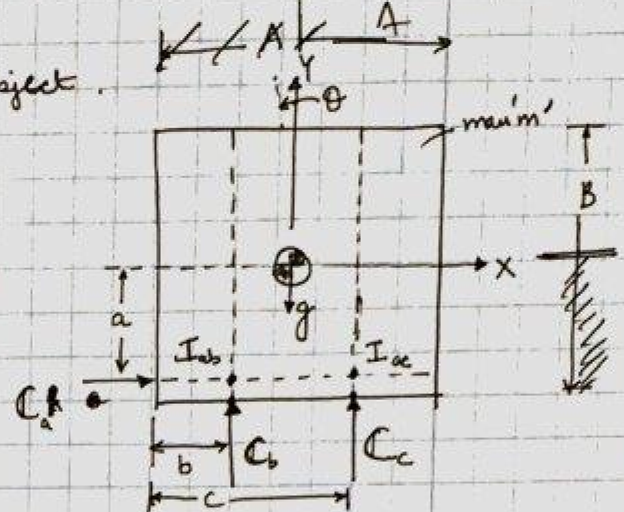
(12) When parts are connected in series (cascaded), add the degrees of freedom. When the connections occur in parallel, add constraints.

Understanding the problem:

Q. How to repeatedly fixture a 2D object.

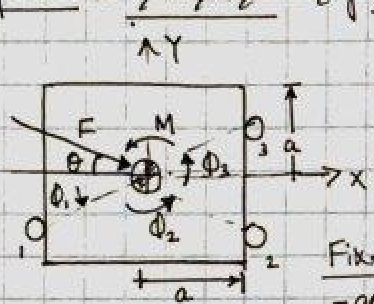
3-dof to define.
→ need 3 contact points.

force → gravity. (Externally applied load to any part of object)

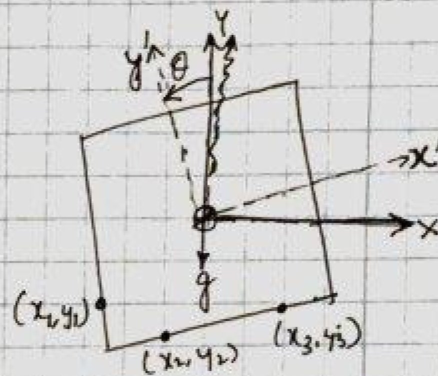


- Center of mass at (0,0).

- inputs: ~~orientation~~ θ ~~of board~~.

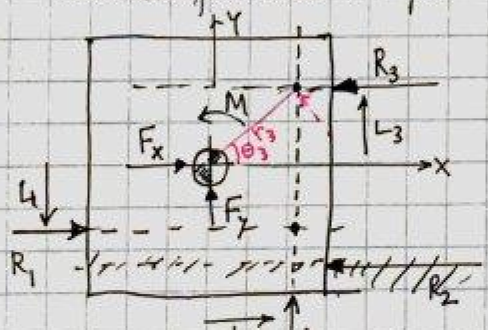


Fixed:
- geo. - square (side 'a')



~~Then~~ Inputs:
- F, M about the center of mass. ✓
- position of pins.

Output
- reaction forces on the pins. ✓



- Reactions R are always normal to the surface at the point of contact.

- ∴ depending on ϕ, select the direction of R.

- if $\phi \in (-45°, 45°)$, R in '-x'
  $\in (45, 135)$, R in '-y'
  $\in (135, 225)$ R in '+x' ($-180, -180$)
  $\in (225, -45)$ R in '+y' (-135)

Equations to solve:

$+\uparrow \Sigma F_y = 0$ $\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_y + R_2 = 0$ $F_x - R_3 + R_1 = 0$

Need comp ⊥ to line from center to pin.

$+\circlearrowleft \Sigma M_{com} = 0 \Rightarrow M + R_1 L_1 + R_2 L_2 + \underbrace{R_3 L_3}_{R_3 \sin\theta_3 r_3} = 0.$

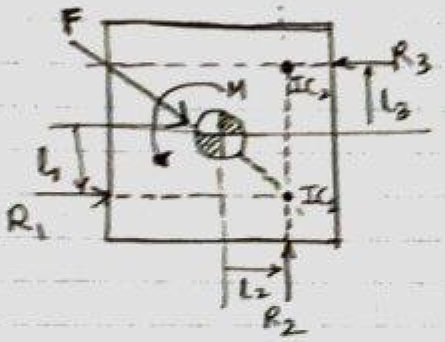
φ Assuming no friction

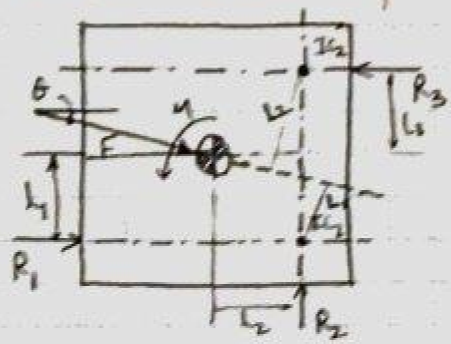
Generalizing,

$$F_y + \uparrow \sum R_{iy} = 0$$
$$F_x + \rightarrow \sum R_{ix} = 0$$
$$M + \sum R_i \cdot L_i = 0$$

Solve to find $\underline{R_i}$.

Using instant centers: very intelligent approach, hard to ([illegible]) analyze geometrically





$\cancel{+\circlearrowleft M_{IC_1} = 0 \Rightarrow M + R_3(L_1 + L_3) = 0 \quad \text{— gives } R_3}$

$\cancel{+\circlearrowleft M_{IC_2} = 0 \Rightarrow M + R_1 \times (L_1 + L_3) = 0 \quad \text{— gives } R_1}$

$$+\circlearrowleft M_{IC_1} = 0 \Rightarrow M + R_3(L_1 + L_3) - F \times L_1 = 0 \quad \text{— gives } R_3$$

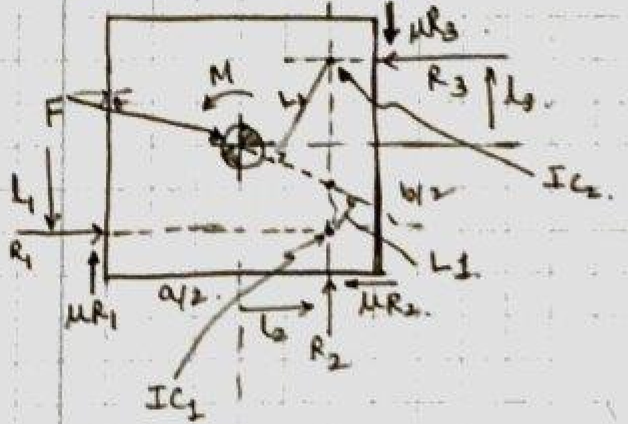
$$+\circlearrowleft M_{IC_2} = 0 \Rightarrow M + R_1(L_1 + L_3) - F \times L_2 = 0 \quad \text{— gives } R_1$$

$$+\uparrow \sum F_y = 0 \Rightarrow -F\sin\theta + R_2 = 0 \quad \text{— gives } R_2.$$

In general –

- Solve for the moment about the I.C.s → find the reacn forces.
- Do a force balance in the direction of the remaining reacn force to find the value of that reaction force.

Model with friction



Now, solve 3 simultaneous linear equations.

$\overset{+}{\rightarrow} \sum F_x = 0$ — frictional force @ R_2 due to applied load

$F\cos\theta + R_1 - R_3 - \mu R_2 = 0$

$+\uparrow \sum F_y = 0$

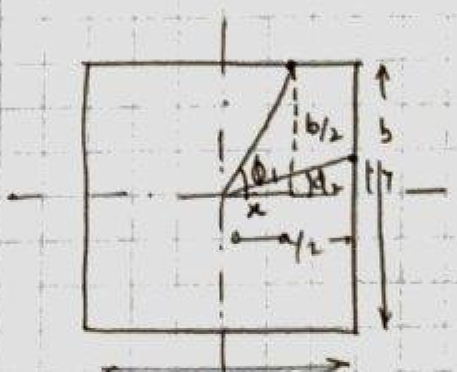
$-F\sin\theta + \mu R_1 - \mu R_3 + R_2 = 0.$

$+\circlearrowleft \sum M_O = 0$

$M + (R_1 l_1 + R_2 l_2 + R_3 l_3) - \mu(R_1 . a/2 + R_2 . b/2 + R_3 . \frac{a}{2}) = 0.$

CAUSED BY F?
OR INDEPENDENT?

l_1, l_2, l_3 can be determined by an angle (say α), which relates the position of the ICs to the C.O.M.



$\frac{y}{a/2} = \tan(\phi_2)$

$\Rightarrow y = \frac{a}{2}\tan(\phi_2)$

$\frac{b/2}{x} = \tan\phi_1$

$x = b/2 / (\tan\phi_1)$

for matrix manipulation:

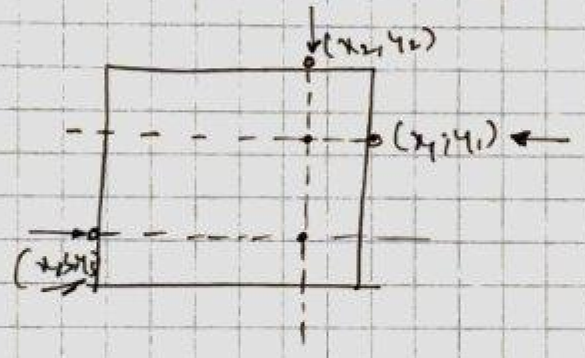
$$F\cos\theta + aR_1 + bR_2 + cR_3 + p\mu R_1 + q\mu R_2 + r\mu R_3 = 0$$

$$F\cos\theta + (a+p\mu)R_1 + (b+q\mu)R_2 + (c+r\mu)R_3 = 0$$

$$\begin{bmatrix} (a_1+p_1\mu) & (b_1+q_1\mu) & (c_1+r_1\mu) \\ (a_2+p_2\mu) & (b_2+q_2\mu) & (c_2+r_2\mu) \\ \cancel{(a_3+p_3\mu)} & \cancel{(b_3+q_3\mu)} & \cancel{(c_3+r_3\mu)} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} -F\cos\theta \\ -F\sin\theta \\ -M \end{bmatrix}$$

$$(m_1 l_1 + n_1 \mu L_1)R_1 + (m_2 l_2 + n_2\mu L_2)R_2 + (m_3 l_3 + n_3 \mu L_3)R_3 = -M.$$

To calculate instant centers:



$$(x_1+\alpha_1)\hat{i} + (y_1+\beta_1)\hat{j}$$
$$(x_2+\alpha_2)\hat{i} + (y_2+\beta_2)\hat{j}$$
$$(x_3+\alpha_3)\hat{i} + (y_3+\beta_3)\hat{j}$$

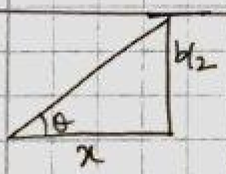
line

$$m = \frac{y_2-y_1}{x_2-x_1}$$

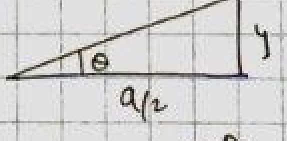
$$y = mx + c$$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow y - y_1 = (x-x_1)m$$

$$y = mx + (y_1 - mx_1)$$



$$\frac{b/2}{x} = \tan\theta \qquad x = \left(\frac{b/2}{\tan\theta}\right)$$



$$\frac{y}{a/2} = \tan\theta$$